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# A GENERALIZED THEORY OF THERMOELASTICITY FOR PRESTRESSED BODIES WITH MICROSTRUCTURE

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**Abstract**—A generalized theory is presented for a thermoelastic dipolar body which has previously received a large deformation and is at nonuniform temperature. A generalized linear theory of dipolar thermoelasticity with initial stress and initial heat flux has been derived.

## NOMENCLATURE

| | |
|---|---|
| $E_{AB}$, $E_{A:B}$, $E_{A:BK}$ | the strain tensors |
| $f_k$ | external body force per unit mass |
| $f_{iK}$ | dipolar body force per unit mass |
| $Q_K$ | the heat flux across surfaces that were originally coordinate planes perpendicular to the $X_K$ axes, measured per unit area of these planes, per unit time |
| $s$ | the heat supply per unit mass and time |
| $S_{KiB}$, $S_{AB}$ | the generalized stress tensors |
| $T$ | absolute temperature |
| $T_{Ki}$ | first Piola–Kirchhoff stress tensor ($T_{Ki} \neq T_{iK}$) |
| $Y_{KL}$ | the inertia coefficients |
| $\Psi$ | Gibbs free energy per unit of initial volume |
| $\rho_O$ | initial mass density |
| $\delta_{AB}$ | Kronecker delta |
| $\eta$ | the entropy per unit mass. |

## 1. INTRODUCTION

During the past two decades, many papers have been devoted to the generalized theory of thermoelasticity, which avoid the paradox of propagation of heat with infinite speed. Some of the papers worth mentioning are by Lord and Shulman (1967), Kaliski and Nowacki (1970), Green and Lindsay (1972), Boschi and Iesan (1973), Dost and Tabarrok (1978), Sherief and Dhaliwal (1980), Smirnov (1981), Lebon (1982), and Chandrasekharaiah (1986). This paradox is produced by the diffusion type equation governing the temperature field in the classical theories.

The purpose of the present paper is to establish a generalized theory for a thermoelastic dipolar body which has been prestressed. The coupled theory for a dipolar prestressed thermoelastic body has been obtained by Iesan (1981).

In Section 2, we modify the nonlinear constitutive equations of dipolar thermoelastic bodies developed by Green and Rivlin (1964), which provide a foundation for our later work. In Section 3, we derive the generalized thermoelastic equations for a dipolar body which has previously received large deformation and is at nonuniform temperature. These results are then used, in Section 4, to derive the basic equations for a special case concerning small thermoelastic deformations in a dipolar body which is under initial stress and initial heat flux.

## 2. NONLINEAR FORMULAE

Consider a thermoelastic dipolar body, whose configuration changes continuously under external mechanical actions and heating, from an original reference state $\Omega_o$ to a deformed state $\Omega$. Let $\partial\Omega_o$ and $\partial\Omega$ be the surfaces of $\Omega_o$ and $\Omega$, respectively. $X_K$ will denote the position of the particle $X$ in the reference state, and $x_k$ the position in the deformed state, where $x_k = x_k(X_1, X_2, X_3, t)$ and $t$ is time.

In the theory of dipolar thermoelastic bodies developed by Green and Rivlin (1964), the deformation is described by the displacement $x_k = x_k(X_1, X_2, X_3, t)$ and dipolar displacement $x_{kA} = x_{kA}(X_1, X_2, X_3, t)$. Let $X_{kA}$ be the value of $x_{kA}$ in the reference state and $X_{BA} = \delta_{kB} X_{kA}$. We assume that $x_k$ and $x_{kA}$ are sufficiently smooth, and that

$$J \equiv \det(\partial x_i/\partial X_A) > 0 \quad \text{and} \quad \det(x_{kA}) \neq 0.$$

In the material description, the basic nonlinear equations in dipolar thermoelasticity take the form:

$$T_{Kk,K} + \rho_o f_k = \rho_o \ddot{x}_k, \tag{1}$$

$$S_{MiK,M} - x_{i,A} S_{AK} + \rho_o f_{iK} = \rho_o Y_{MK} \ddot{x}_{iM}, \tag{2}$$

$$\rho_o T \dot{\eta} = \rho_o s - Q_{A,A} \tag{3}$$

in $\Omega_o \times [0, t_o)$, where $t_o$ is some time instant that may be infinite.

The constitutive variables are $E_{AB}$, $E_{A:B}$, $E_{A:BK}$, $T$ and $T_{,K}$ with the geometrical equations

$$2E_{AB} = x_{i,A} x_{i,B} - \delta_{AB} \tag{4}$$

$$E_{A:B} = x_{i,B} x_{iA} - X_{BA}, \tag{5}$$

$$E_{A:BK} = x_{i,B} x_{iA,K} - X_{BA,K} \tag{6}$$

and the constitutive equations are

$$P_{AB} = X_{A,i}(T_{Bi} - S_{BC} x_{iC} - T_{KBC} x_{iC,K}) = P_{BA} = \partial\Psi/\partial E_{AB}, \tag{7}$$

$$T_{KAB} = X_{A,i} S_{KiB} = \partial\Psi/\partial E_{BA:K}, \tag{8}$$

$$s_{AB} = \partial\Psi/\partial E_{B:A}, \tag{9}$$

$$\eta = -\frac{1}{\rho_o}\partial\Psi/\partial T, \quad \partial\Psi/\partial T_{,K} = 0. \tag{10}$$

A superposed dot stands for the material time derivative while a comma followed by a subscript denotes partial derivatives with respect to the spatial coordinates, in the deformed configuration if the subscript is a minuscule, in the reference configuration if the subscript is a majuscule; that is, $g_{,A} = \partial g/\partial X_A$ and $g_{,k} = \partial g/\partial x_k$. Einstein summation on repeated indices is also used throughout this paper.

We assume that $\Psi$, $T_{KL}$, $\eta$, $Q_K$ are written as symmetric functions of $E_{KL}$ in the indices $K$ and $L$, and $E_{KL}$ is understood to mean $(E_{KL} + E_{LK})/2$ in $\partial\Psi/\partial E_{KL}$.

We assume that the heat flux $Q_K$ satisfies a first order rate equation of the form [see Chandrasekharaiah (1986) and Müller (1967)]:

$$\dot{Q}_K = \mathscr{L}_K(E_{AB}, E_{A:B}, E_{A:BL}, T, T_{,L}, Q_K), \tag{11}$$

with $\partial\dot{Q}_K/\partial Q_L = 0$ if $K \neq L$. Since it is required that $Q_K$ vanishes at equilibrium, i.e. for $T_{,L} = 0$, we must also have that

$$\mathscr{L}_K(E_{AB}, E_{A:B}, E_{A:BL}, T, 0, 0) = 0. \quad (12)$$

## 3. SMALL DEFORMATION SUPERPOSED ON A LARGE DEFORMATION

In this section, we consider three states of the body, the initial state $\Omega_o$, the first deformed state $\Omega$, and the final deformed state $\Omega^*$ corresponding respectively to temperature $T_o$ and zero displacement; temperature $T$ and displacement $\mathbf{v}$; and temperature $T^*$ and displacement $\mathbf{v}+\mathbf{u}$. Following Iesan (1981), we call $\Omega$ the primary state and $\Omega^*$ the secondary state. Thermodynamic quantities and forces associated with $\Omega^*$ will be denoted with an asterisk. The position coordinates of the particle $X_A$ at time $t$ in $\Omega^*$ will be denoted by $y_i = y_i(X_1, X_2, X_3, t)$ and $y_{iA} = y_{iA}(X_1, X_2, X_3, t)$ with

$$J^* = \det(y_{i,A}) > 0 \quad \text{and} \quad \det(y_{iA}) \neq 0.$$

We shall define the incremental displacement $u_i$, incremental dipolar displacement $u_{iA}$ and incremental temperature $\theta$, respectively, as

$$u_i = y_i - x_i, \quad u_{iA} = y_{iA} - x_{iA}, \quad \theta = T^* - T. \quad (13)$$

Here we consider the case in which $u_i, u_{iA}$ and $\theta$ are small. Suppose that there exists a real parameter $\varepsilon$, small enough for its squares and higher powers to be neglected, such that

$$u_i = \varepsilon u'_i, \quad u_{iA} = \varepsilon u'_{iA}, \quad \theta = \varepsilon\theta', \quad (14)$$

where $u'_i$, $u'_{iA}$ and $\theta'$ are independent of $\varepsilon$.

In the secondary state $\Omega^*$, the basic equations are of the same form as those in the primary state $\Omega$. We have the equations of motion

$$T^*_{Kk,K} + \rho_o f^*_k = \rho_o \ddot{y}_k \quad (15)$$

$$S_{MiK,M} - x_{i,A} S_{AK} + \rho_o f_{iK} = \rho_o Y_{MK} \ddot{x}_{iM}, \quad (16)$$

the energy equation

$$\rho_o T^* \dot{\eta}^* = \rho_o s^* - Q^*_{A:A}, \quad (17)$$

the geometrical equations

$$2E^*_{AB} = y_{i,A} y_{i,B} - \delta_{AB}, \quad (18)$$

$$E^*_{A:B} = y_{i,B} y_{iA} - X_{BA}, \quad (19)$$

$$E^*_{A:BK} = y_{i,B} y_{iA,K} - X_{BA:K} \quad (20)$$

and the constitutive equations are

$$P^*_{AB} = \frac{\partial X_A}{\partial y_i}(T^*_{Bi} - S^*_{BC} y_{iC} - T^*_{KBC} y_{iC,K}) = P^*_{BA} = \partial\Psi^*/\partial E^*_{AB}, \quad (21)$$

$$T^*_{KAB} = \frac{\partial X_A}{\partial y_i} S^*_{KiB} = \partial\Psi^*/\partial E^*_{BA:K}, \quad (22)$$

$$S^*_{AB} = \partial\Psi^*/\partial E^*_{B:A}, \quad (23)$$

$$\eta^* = -\frac{1}{\rho_o}\partial\Psi^*/\partial T^*, \quad \partial\Psi^*/\partial T^*_{,K} = 0, \tag{24}$$

$$\dot{Q}^*_K(X_K, t) = \mathscr{L}_K(E^*_{AB}, E^*_{A:B}, E^*_{A:BL}, T^*, T^*_{,L}, Q^*_K). \tag{25}$$

Now we shall derive the equations satisfied by $u_i$, $u_{iA}$ and $\theta$. Keeping (14) in mind, we get the following second order approximation:

$$\begin{aligned}
&\Psi(E^*_{AB}, E^*_{A:B}, E^*_{A:BK}, T^*) - \Psi(E_{AB}, E_{A:B}, E_{A:BK}, T) \\
&\quad = a\theta + c_{AB}(E^*_{AB} - E_{AB}) + c_{A:B}(E^*_{A:B} - E_{A:B}) + c_{A:BK}(E^*_{A:BK} - E_{A:BK}) \\
&\qquad - \tfrac{1}{2}d\theta^2 - B_{AB}\theta(E^*_{AB} - E_{AB}) - B_{A:B}\theta(E^*_{A:B} - E_{A:B}) - B_{A:BK}\theta(E^*_{A:BK} - E_{A:BK}) \\
&\qquad + \tfrac{1}{2}D_{ABCD}(E^*_{AB} - E_{AB})(E^*_{C:D} - E_{C:D}) + \tfrac{1}{2}D_{ABCDK}(E^*_{AB} - E_{AB})(E^*_{C:DK} - E_{C:DK}) \\
&\qquad + \tfrac{1}{2}G_{ABCDK}(E^*_{A:B} - E_{A:B})(E^*_{C:DK} - E_{C:DK}) + \tfrac{1}{2}C_{ABCD}(E^*_{AB} - E_{AB})(E^*_{CD} - E_{CD}) \\
&\qquad + \tfrac{1}{2}C'_{ABCD}(E^*_{A:B} - E_{A:B})(E^*_{C:D} - E_{C:D}) \\
&\qquad + \tfrac{1}{2}C_{ABKLMN}(E^*_{A:BK} - E_{A:BK})(E^*_{L:MN} - E_{L:MN}),
\end{aligned} \tag{26}$$

where $B_{AB}$, $C_{ABCD}$, $C'_{ABCD}$ and $C_{ABKLMN}$ have certain symmetries in their indices, and that

$$a = \partial\Psi/\partial T = -\rho_o\eta, \quad c_{AB} = \partial\Psi/\partial E_{AB} = P_{AB}, \tag{27}$$

$$c_{A:B} = \partial\Psi/\partial E_{A:B} = S_{BA}, \quad c_{A:BK} = \partial\Psi/\partial E_{A:BK} = T_{KBA}. \tag{28}$$

From eqns (21)–(24), (26)–(28), we find that

$$P^*_{AB} = P_{AB} - B_{AB}\theta + D_{ABCD}(E^*_{C:D} - E_{C:D}) + D_{ABCDK}(E^*_{C:DK} - E_{C:DK}) + C_{ABCD}(E^*_{CD} - E_{CD}), \tag{29}$$

$$S^*_{BA} = S_{BA} - B_{A:B}\theta + D_{ABCD}(E^*_{CD} - E_{CD}) + G_{ABCDK}(E^*_{C:DK} - E_{C:DK}) + C'_{ABCD}(E^*_{C:D} - E_{C:D}), \tag{30}$$

$$\begin{aligned}
T^*_{KBA} = T_{KBA} - B_{A:BK}\theta + D_{CDABK}(E^*_{CD} - E_{CD}) \\
+ G_{CDABK}(E^*_{C:D} - E_{C:D}) + C_{ABKLMN}(E^*_{L:MN} - E_{L:MN}),
\end{aligned} \tag{31}$$

$$\rho_o\eta^* = \rho_o\eta + \mathrm{d}\theta + B_{AB}(E^*_{AB} - E_{AB}) + B_{A:B}(E^*_{A:B} - E_{A:B}) + B_{A:BK}(E^*_{A:BK} - E_{A:BK}). \tag{32}$$

Also, from eqns (11) and (25), we have

$$\begin{aligned}
\dot{Q}^*_K - \dot{Q}_K = -\frac{1}{\tau}\{(Q^*_K - Q_K) + a_K\theta + k_{KL}\theta_{,L} + h_{KMN}(E^*_{MN} - E_{MN}) \\
+ h'_{KMN}(E^*_{M:N} - E_{M:N}) + h_{KMNL}(E^*_{M:NL} - E_{M:NL})\}.
\end{aligned} \tag{33}$$

From eqn (12), we find that

$$a_K = 0, \quad h_{KMN} = 0, \quad h'_{KMN} = 0, \quad h_{KMNL} = 0, \quad \text{if} \quad T = \text{constant}. \tag{34}$$

From eqns (4)–(6) and (18)–(20), and the relations

$$y_{i,A} = x_{i,A} + u_{i,A}, \quad y_{iA,B} = x_{iA,B} + u_{iA,B}, \tag{35}$$

we find that

$$2E^*_{AB} = 2E_{AB}+x_{i,A}u_{i,B}+x_{i,B}u_{i,A}, \quad (36)$$

$$E^*_{A:B} = E_{A:B}+x_{iA}u_{i,B}+x_{i,B}u_{iA}, \quad (37)$$

$$E^*_{A:BC} = E_{A:BC}+x_{iA,C}u_{i,B}+x_{i,B}u_{iA,C}, \quad (38)$$

where we have used the fact that $u_i$s and $u_{iB}$s are small and hence the terms like $u_{i,A}u_{i,B}$ have been neglected. Let

$$e_{AB} = \tfrac{1}{2}(x_{i,A}u_{i,B}+x_{i,B}u_{i,A}), \quad (39)$$

$$v_{AB} = x_{iA}u_{i,B}+x_{i,B}u_{iA}, \quad (40)$$

$$v_{ABC} = x_{iA,C}u_{i,B}+x_{i,B}u_{iA,C}, \quad (41)$$

then we have

$$E^*_{AB} = E_{AB}+e_{AB}, \quad (42)$$

$$E^*_{A:B} = E_{A:B}+v_{AB}, \quad (43)$$

$$E^*_{A:BC} = E_{A:BC}+v_{ABC}. \quad (44)$$

Substituting from eqns (42)–(44) into eqns (29)–(33), we find that

$$P^*_{AB} = P_{AB}+J^{(1)}_{AB}, \quad (45)$$

$$S^*_{BA} = S_{BA}+J^{(2)}_{BA}, \quad (46)$$

$$T^*_{KBA} = T_{KBA}+J^{(3)}_{BAK}, \quad (47)$$

$$\rho_o\eta^* = \rho_o\eta+J^{(4)}, \quad (48)$$

$$Q^*_K-Q_K+\tau(\dot{Q}^*_K-\dot{Q}_K) = -a_K\theta-k_{KL}\theta_{,L}-h_{KMN}e_{MN}-h'_{KMN}v_{MN}-h_{KMNL}v_{MNL}, \quad (49)$$

where

$$J^{(1)}_{AB} = -B_{AB}\theta+D_{ABCD}v_{CD}+D_{ABCDK}v_{CDK}+C_{ABCD}e_{CD}, \quad (50)$$

$$J^{(2)}_{BA} = -B_{A:B}\theta+D_{ABCD}e_{CD}+G_{ABCDK}v_{CDK}+C'_{ABCD}v_{CD}, \quad (51)$$

$$J^{(3)}_{BAK} = -B_{A:BK}\theta+D_{CDABK}e_{CD}+G_{CDABK}v_{C:D}+C_{ABKLMN}v_{LMN}, \quad (52)$$

$$J^{(4)} = \mathrm{d}\theta+B_{AB}e_{AB}+B_{A:B}v_{AB}+B_{A:BK}v_{ABK}. \quad (53)$$

From eqns (7), (8), (21), (22) and (45)–(47), we get

$$T^*_{Bi} = T_{Bi}+u_{i,A}P_{AB}+u_{iC}S_{BC}+u_{iC,K}T_{KBC}+x_{i,A}J^{(1)}_{AB}+x_{iC}J^{(2)}_{BC}+x_{iC,K}J^{(3)}_{CBK}, \quad (54)$$

$$S^*_{KiB} = S_{KiB}+u_{i,A}T_{KAB}+x_{i,A}J^{(3)}_{BAK}. \quad (55)$$

If we denote

$$\pi_{Bi} = T^*_{Bi} - T_{Bi}, \quad N_{AB} = S^*_{AB} - S_{AB}, \quad M_{KkM} = S^*_{KkM} - S_{KkM},$$

$$\Phi_K = Q^*_K - Q_K, \quad \gamma = \eta^* - \eta,$$

then we have

$$\pi_{Bi} = u_{i,A}P_{AB} + u_{iC}S_{BC} + u_{iC,K}T_{KBC} + x_{i,A}J^{(1)}_{AB} + x_{iC}J^{(2)}_{BC} + x_{iC,K}J^{(3)}_{CBK}, \tag{56}$$

$$N_{BA} = J^{(2)}_{BA}, \tag{57}$$

$$M_{KiB} = u_{i,A}T_{KAB} + x_{i,A}J^{(3)}_{BAK}, \tag{58}$$

$$\rho_o \gamma = J^{(4)}, \tag{59}$$

$$\Phi_K + \tau\dot{\Phi}_K = -a_K\theta - k_{KL}\theta_{,L} - h_{KMN}e_{MN} - h'_{KMN}v_{MN} - h_{KMNL}v_{MNL}. \tag{60}$$

Subtracting eqns (1), (2) and (3) from eqns (15), (16) and (17), respectively, we find the incremental equations to be

$$\pi_{Ki,K} + \rho_o F_i = \rho_o \ddot{u}_i, \tag{61}$$

$$M_{MiK,M} - x_{i,A}N_{AK} + \rho_o F_{iK} = \rho_o Y_{MK}\ddot{u}_{iM}, \tag{62}$$

$$\rho_o T\dot{\gamma} + \rho_o \theta\dot{\eta} = -\Phi_{K,K} + \rho_o S, \tag{63}$$

where

$$F_i = f^*_i - f_i, \quad F_{iK} = f^*_{iK} - f_{iK} \quad \text{and} \quad S = s^* - s. \tag{64}$$

## 4. LINEAR GENERALIZED THEORY OF PRESTRESSED DIPOLAR THERMOELASTICITY

In this section, we consider the special case when the primary state $\Omega$ of the body is identical with that of the initial body $\Omega_o$ so that $x_1 = X_1, x_2 = X_2, x_3 = X_3$, and we suppose that $\Omega_o$ is subjected to an initial stress and an initial heat flux caused by the nonuniform initial temperature $T_o$. Due to the action of external loadings and heat sources, the body $\Omega_o$ undergoes a deformation. There arise displacements $u_i = \varepsilon u'_i$, $u_{iK} = \varepsilon u'_{iK}$, and temperature acquires an increment $\theta = \varepsilon\theta'$. Here we systematically neglect all powers of $\varepsilon$ above the first, except in the free energy function $\Psi$ where we retain quadratic terms in $\varepsilon$.

The work of previous section can be applied immediately to this special case and yields a generalized linear thermoelasticity theory in the presence of initial stress and heat flux. In this case, we have

$$x_{i,A} = \delta_{iA}, \quad J = 1, \quad T = T_o, \quad \dot{\eta} = 0,$$

$$E_{AB} = 0, \quad E_{A:B} = 0, \quad E_{A:BK} = 0.$$

Obviously, all coefficients defined in the previous section are now evaluated at $E_{KL} = 0$, $E_{A:B} = 0$, $E_{A:BK} = 0$, and $T = T_o$. From eqns (39)–(41), we find that

$$e_{AB} = \tfrac{1}{2}(u_{A,B} + u_{B,A}), \tag{65}$$

$$v_{AB} = u_{AB} + X_{CA}u_{C,B}, \tag{66}$$

$$v_{ABC} = X_{KA,C}u_{K,B} + u_{BA,C}. \tag{67}$$

The governing equations (61)–(63) become

$$\pi_{KA,K}+\rho_o F_A = \rho_o \ddot{u}_A, \tag{68}$$

$$M_{MAK,M}-N_{AK}+\rho_o F_{AK} = \rho_o Y_{MK}\ddot{u}_{AM}, \tag{69}$$

$$\rho_o T_o \dot{\gamma} = -\Phi_{K,K}+\rho_o S. \tag{70}$$

The constitutive relations (56)–(60) reduce to

$$\pi_{BK} = u_{K,A}P_{AB}+u_{KC}S_{BC}+u_{KC,N}T_{NBC}+J^{(1)}_{KB}+X_{KC}J^{(2)}_{BC}+X_{KC,N}J^{(3)}_{CBN}, \tag{71}$$

$$N_{BA} = J^{(2)}_{BA}, \tag{72}$$

$$M_{KNB} = u_{N,A}T_{KAB}+J^{(3)}_{BNK}, \tag{73}$$

$$\rho_o \gamma = J^{(4)}, \tag{74}$$

$$\Phi_K+\tau\dot{\Phi}_K = -a_K\theta-k_{KL}\theta_{,L}-h_{KMN}e_{MN}-h'_{KMN}v_{MN}-h_{KMNL}v_{MNL}. \tag{75}$$

The functions $P_{AB}$, $S_{BC}$ and $T_{NBC}$ characterize the initial stress and they can be arbitrary functions apart from satisfying the energy equation for the static case and the condition that $\Omega_o$ is in equilibrium. The presence of the initial heat flux vector is determined by the nonuniformity of the initial temperature $T_o$. This fact implies the appearance of the coefficients $a_K$, $h_{KMN}$, $h'_{KMN}$ and $h_{KMNL}$ in the expression for $\Phi_A$. If $T_o$ is constant, then $Q_A = 0$, $a_K = 0$, $h_{KMN} = 0$, $h'_{KMN} = 0$ and $h_{KMNL} = 0$.

Substituting from eqns (74) and (75) into the heat conduction equation (70), we find that

$$T_o\left(1+\tau\frac{\partial}{\partial t}\right)(\mathrm{d}\dot{\theta}+B_{AB}\dot{e}_{AB}+B_{A:B}\dot{v}_{AB}+B_{A:BK}\dot{v}_{ABK})$$

$$= (a_K\theta+k_{KL}\theta_{,L}+h_{KMN}e_{MN}+h'_{KMN}v_{MN}+h_{KMNL}v_{MNL})_{,K}+\rho_o\left(1+\tau\frac{\partial}{\partial t}\right)S, \tag{76}$$

which may be a hyperbolic equation if the coefficients have proper signs. As a consequence, heat will propagate at a finite speed and the paradox of infinite speed is thus circumvented.

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